# Weakly First Order Cosmological Phase Transitions and Fermion Production

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We study weakly first order cosmological phase transitions in finite temperature field theories. Focusing on the standard electroweak theory and its minimal supersymmetric extension, we identify the regimes of Higgs masses for which the phase transition in these models proceeds by significant phase mixing and the coarsening of the subsequent domain network. This dynamics is distinct from that for strongly first order transitions, which proceed by the nucleation and propagation of critical bubbles. We describe how electroweak baryogenesis might take place in these models, explaining how our new picture can relax the sphaleron washout bound of traditional scenarios.

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Finite temperature phase transitions are of great interest in cosmology because they provide a mechanism by which remnants of the early universe can make an observable imprint on today's universe. Well-known examples of this are cosmological inflation, the production of topological defects, and electroweak baryogenesis [1].

Cosmological phase transitions are well-understood in two particular limits. Strongly first order phase transitions occur when a system begins in a metastable state that is separated from a global minimum by a sufficiently large energy barrier. In such a system, widely separated points in space undergo quantum tunneling or thermal hopping events, in which bubbles of the true vacuum nucleate in the background sea of false vacuum: bubbles which are sufficiently large that their volume energy dominates over their surface tension expand and eventually coalesce, completing the phase transition. If the phase transition is second order (continuous), the system begins in an unstable state, with no energy barrier separation from the global minimum. Small-amplitude, large-wavelength fluctuations grow, followed by domain coarsening and phase separation. This is referred to as spinodal decomposition [2].

Away from these two extreme limits, the dynamics of phase transitions is much less well understood. Numerical simulations provide accurate estimates of the strength of the phase transition, and determine the point at which first order transitions become second order as parameters of the theory are varied, as recently done for the standard model (SM) [3]. However, for weakly first order phase transitions an understanding of *how* the phase transition proceeds and completes is far more vague.

It is widely believed that, in the absence of exotic means for departing from equilibrium, a strongly first order phase transition is required for electroweak baryogenesis to occur.(For a recent review see [4].) This is for two main reasons. If bubble nucleation is the mechanism, then the sharp change in order parameter across the bubble walls provides a large departure from equilibrium at each point in space swept out by the walls. Further, the associated large energy barrier between phases leads to thermal anomalous fermion number violation being sufficiently suppressed in the true vacuum so that any baryon number produced is not washed out [5]. For a weakly first order transition this is not the case, and the traditional criterion that the transition be sufficiently strong provides a constraint on the theory. This constraint can be expressed as $\langle\phi(T_c)\rangle/T_c \geq 1$, where $\phi$ is the Higgs field, and $T_c$ is the critical temperature for the electroweak phase transition. This translates directly into a bound on the mass of the Higgs boson, which, for the minimal supersymmetric standard model (MSSM) is $m_H^{(\rm MSSM)} < 105$ GeV [6].

In this letter, we investigate an alternative scenario for fermion production in weakly first order phase transitions. When transitions are weak, large-amplitude subcritical fluctuations between the symmetric and broken-symmetric phases cause the transition to begin by significant phase mixing and, after inter-phase fluctuations cease, to proceed by coarsening of the subsequent domain network. The crucial point is that the sphaleron washout temperature need not be the critical temperature, but the lower temperature when the large-amplitude fluctuations freeze out, the *Ginzburg temperature*, $T_G$. Thus, fermion production and preservation may still be efficient in this picture, and possibly remain sufficient to account for the baryon asymmetry of the universe. Let us describe how this might happen using the SM and the more promising MSSM as examples.

The finite temperature effective potential for the magnitude of the Higgs field in electroweak models can often be written as

$$V(\phi,T) = D(T^2 - T_2^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4 \ , \qquad (1)$$

with $T_2^2 = (m_H^2 - 8Bv^2)/4D$, where, for the SM,

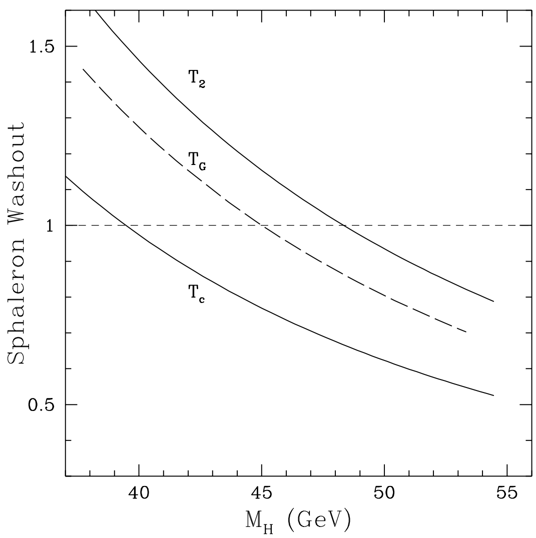
$$D_{\rm SM} = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2} \ ,$$

$$E_{\rm SM} = \frac{2m_W^3 + m_Z^3}{4\pi v^3} \ ,$$

$$B_{\rm SM} = \frac{3}{64\pi^2 v^4}(2m_W^4 + m_z^4 - 4m_t^4) \ , \qquad (2)$$

and $\lambda(T)$ is the temperature-corrected Higgs coupling. It is meaningful to speak of there being a global minimum, separated from a metastable minimum by an energy barrier, for $T_2^2 < T^2 < T_c^2 \equiv \frac{\lambda D}{\lambda D - E^2} T_2^2$. Let us begin by examining how the sphaleron washout temperature varies in this range, for changing Higgs mass. In figure (1) we plot $\langle\phi(T)\rangle/T$ as a function of the Higgs mass $m_H$, for three values of the temperature, $T_c$, $T_G$, and $T_2$.

FIG. 1. Standard Model Washout.



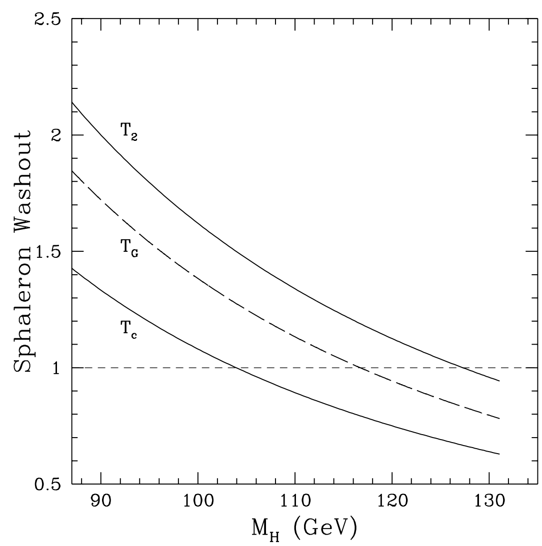

The lower curve on this figure demonstrates how the the traditional sphaleron bound arises, since this curve crosses unity at a Higgs mass around 40 GeV. However, this, along with the upper curve shows that, for Higgs masses in the range 40 GeV $< m_H^{\rm SM} <$ 48 GeV, the temperature below which sphaleron washout is avoided may be reached before the barrier between phases has vanished. This is the relevant regime for our scenario.

Although the SM is useful for illustrating this effect, experimental bounds place the physical Higgs mass at $m_H > 100$ GeV. Thus, our analysis is academic in this model. However, in the MSSM, we shall see that useful results are obtained. For calculational convenience, in this letter we study the MSSM in the limit in which the extra Higgs fields and all the sparticles are decoupled from the spectrum. Thus, all that remains is the lightest Higgs and the right-handed stop. In this limit, the one-loop finite-temperature effective potential can be written in the same form as the SM (1), but with the following parameters

$$D_{\rm MSSM} = \frac{2m_W^2 + m_Z^2 + 4m_t^2}{8v^2} \ ,$$
$$E_{\rm MSSM} = \frac{2m_W^3 + m_Z^3 + 2m_t^3}{4\pi v^3} \ ,$$
$$B_{\rm MSSM} = \frac{3}{64\pi^2 v^4}(2m_W^4 + m_z^4 - 2m_t^4) \ . \qquad (3)$$

For this potential, we can again examine how the sphaleron washout temperature varies with Higgs mass. We note that two-loop corrections to the MSSM potential have been extensively discussed in the literature. In fact, the consensus is that the inclusion of two-loop effects enhances the strength of the transition, further opening the window of allowed Higgs masses for efficient baryogenesis [7]. As our present purpose is to illustrate a new possible mechanism for baryogenesis within weak first-order phase transitions, we will for simplicity restrict ourselves to one-loop corrections here, since there will always be a range of Higgs masses for which the transition will be weakly first order. However, we are presently working to extending our analysis to include two-loop corrections. We plot our results for the one-loop MSSM potential in figure (2).

FIG. 2. Minimal Supersymmetric Standard Model Washout.




Notice that, for the MSSM, there is a range of masses for the lightest Higgs, within which the phase transition is weakly first order, but with the possibility for sphaleron transitions to become suppressed in the broken phase before the end of the transition.

The phase transition begins at the critical temperature, $T_c$. At this temperature, when the phase transition is sufficiently weakly first order (the sphaleron bound is violated at $T_c$ but not at $T_G$), there are significant subcritical fluctuations, both of the broken-symmetric (the true vacuum, +) and the symmetric one (false vacuum, 0). Consequently, significant phase mixing occurs and homogeneous nucleation breaks down [8], [9]. We write the rates of the relevant subcritical fluctuations as $G_{0\to+}$ and $G_{+\to0}$, respectively. As the phase-mixed plasma cools due to the expansion of the universe, it reaches the tem-

perature $T_G$, at which fluctuations back to the false vacuum freeze out. The criterion for this to occur is

$$\left.\frac{G_{+\to 0}}{H}\right|_{T_G} = 1 \ , \tag{4}$$

where $H$ is the Hubble parameter. The question of how the phase transition proceeds after this depends on how the fraction, $\gamma_+(T_G)$, of the total volume in the broken phase at the Ginzburg temperature compares with the percolation threshold, $p_{\rm perc} \simeq 0.31$, in three dimensions. If $\gamma_+(T_G) < p_{\rm perc}$, we refer to *partial phase mixing* initially. In this case, isolated domains of the + phase will grow due to pressure difference. However, if $\gamma_+(T_G) > p_{\rm perc}$, then we have *total phase mixing* initially, and both phases percolate, being separated by a convoluted wall, plus small isolated domains of each phase.

For both scenarios, baryon-number production will depend on the ratio $\langle\phi(T_G)\rangle/T_G$, which is shown in figures (1) and (2) as the (central) dashed line. Clearly, the details of how baryon number will percolate into the broken-symmetric phase is determined by how $\gamma_+(T_G)$ compares to $p_{\rm perc}$, that is, on whether the broken-symmetric phase has percolated or not. However, our focus here is on what we could call the necessary condition for baryogenesis to occur, which is fixed by the ratio $\langle\phi(T_G)\rangle/T_G$, irrespective of the details of the phase transition. There are two possibilities, described as follows: If this ratio is larger than one, baryon-number violation is suppressed in the broken phase at freeze-out ($T_G$) and a net baryonic excess will be generated as domains of the broken phase advance on the symmetric phase. If this ratio is smaller than one, any baryon excess will be erased in the broken phase until the temperature drops so that the ratio $\langle\phi(T)\rangle/T$ reaches unity, say at a temperature $\bar{T}$. Thus, in this second case we may expect a severe (if not deadly) volume-suppression factor, unless the domains advance sufficiently slowly so that $T_G \gtrsim \bar{T}$: if $T_G$ were not very close to $\bar{T}$, the domains of the symmetric phase – the sources of baryon-number excess, would have plenty of time to shrink into oblivion before $\bar{T}$ could be reached.

Clearly, we require an estimate of $T_G$ and $\gamma_+(T_G)$. To proceed, we assume that the fluctuations are spherical, and consider fluctuations of amplitude $\phi_{0c}$ ($\phi_{+c}$) and spatial size, $R \geq \xi$, the correlation length, with profiles

$$\begin{aligned} \phi_{0\to c}(r) &= \phi_{0c} e^{-r^2/R^2} \ , \\ \phi_{+\to c}(r) &= (\phi_{+c} - \phi_+) e^{-r^2/R^2} + \phi_+ \ . \end{aligned} \tag{5}$$

This *ansatz* has been shown to be *quantitatively* in agreement with numerical simulations [10]. These fluctuations are drawn from a Gibbs distribution,

$$G_{\substack{+\to 0 \\ 0\to +}} = A \exp\left(-\frac{F_{\substack{+\\0}}}{T}\right) \ , \tag{6}$$

where $F_{\substack{+\\0}}$ is the free energy of the respective fluctuation. The prefactor $A$ is expected to be small, within the range $1 \leq A \leq 100$, although its calculation for subcritical bubble nucleation is still an open question. A given fluctuation $\phi$ away from a local minimum of the free-energy density has a free energy "cost"

$$F(\phi) = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi)\right] \ . \tag{7}$$

With the *ansatz* above, these free energies take the form

$$F_{\substack{+\\0}} = \alpha_{\substack{+\\0}} R + \beta_{\substack{+\\0}} R^3 \ , \tag{8}$$

where,

$$\begin{aligned} \alpha_0(\phi) &= \frac{3\pi\sqrt{2\pi}}{8}\phi_{0c}^2 \ , \\ \beta_0(\phi) &= \pi^{3/2}\phi_{0c}^2\left[\frac{\sqrt{2}}{4}D(T^2 - T_2^2) - \frac{\sqrt{3}}{9}ET\phi_{0c} + \frac{\lambda}{32}\phi_{0c}^2\right] \ , \end{aligned} \tag{9}$$

$$\begin{aligned} \alpha_+(\phi) &= \frac{3\pi\sqrt{2\pi}}{8}(\phi_{+c} - \phi_+)^2 \ , \\ \beta_+(\phi) &= \pi^{3/2}(\phi_{+c} - \phi_+)\left[\frac{\sqrt{2}}{4}C_1(\phi_{+c} - \phi_+)\right. \\ &\quad \left. + \ \frac{\sqrt{3}}{9}C_2(\phi_{+c} - \phi_+)^2 + \frac{\lambda}{32}(\phi_{+c} - \phi_+)^3\right] \ , \end{aligned} \tag{10}$$

with $C_1 \equiv [D(T^2 - T_2^2) - 3ET\phi_+ + (3/2)\lambda\phi^2]$, and $C_2 \equiv (-ET + \lambda\phi_+)$.

The correlation length in the broken phase, $\xi_+^2 \equiv [V''(\phi_+)]^{-1}$, is

$$\frac{1}{\xi_+^2} = 4D(T^2 - T_2^2)\left[\zeta^2\left(1 + \sqrt{1 - \frac{1}{\zeta^2}}\right) - 1\right] \ , \tag{11}$$

where $\zeta^2 \equiv 9E^2T^2/[8\lambda D(T^2 - T_2^2)] > 1$. Thus, with (9), (10), and (11), we may calculate $T_G$ via (4).

What remains is to compute $\gamma_+(T_G)$. We proceed by considering the Boltzmann equation for the production of bubbles of the two competing phases, a procedure valid below percolation. This is given by [10]:

$$\frac{\partial f_+^d}{\partial t} + 3\frac{\dot{a}}{a}f_+^d = -|v|\frac{\partial f_+^d}{\partial R} + (1 - \gamma_+)G_{0\to +} - \gamma_+ G_{+\to 0} \ , \tag{12}$$

where $v$ is the shrinking velocity of the subcritical bubbles, $a(t)$ is the cosmic expansion factor, and the volume-fraction in the broken phase is given by

$$\gamma_+(t) \simeq \int_{\phi_{\rm max}}^\infty \int_{R_{\rm min}}^\infty \left(\frac{4\pi R^2}{3}\right) f_+^d(R, \phi, t)\, d\phi dR \ , \tag{13}$$

where $f_+^d \equiv \partial n_+/\partial R \partial \phi$ is the distribution function of disconnected domains (hence the $d$) of the broken phase modeled by the subcritical bubbles, and $n_+$ is their number density. This equation can be integrated by defining the quantity $Y_+ \equiv f_+^d/s$, where $s = \frac{2\pi^2}{45} g_* T^3$, is the entropy density, and looking for equilibrium solutions $\dot{Y}_+ = 0$. The solution can be written as

$$\gamma_+ = \frac{I_1(\phi_{\max}, R_{\min})}{1 + I_1(\phi_{\max}, R_{\min}) + I_2(\phi_{\max}, R_{\min})} \ , \tag{14}$$

where

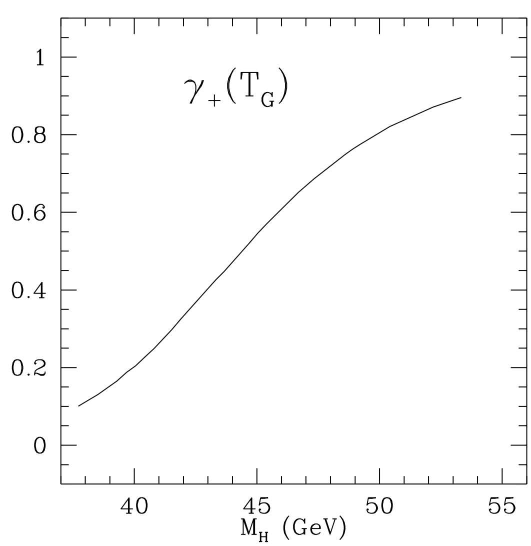
$$I_{\frac{1}{2}}(\phi_{\max}, R_{\min}) \equiv \frac{1}{|v|} \int_{\phi_{\max}}^{\infty} \int_{R_{\min}}^{\infty} \int_{R}^{\infty} d\phi dR dR' \left(\frac{4\pi}{3}\right) R^3 G_{\substack{0 \to + \\ + \to 0}}(R', \phi) \ . \tag{15}$$

Using (5)-(8), we get an expression for the volume fraction $\gamma_+(T_G)$. An analytical solution can be obtained if we neglect the cubic term in (8), which is a good approximation for small enough bubbles, far from percolation. The remaining integrals are then Gaussian, and can be performed to give

$$I_1(\phi_{\max}, R_{\min}, R_{\max}) = \frac{4\pi}{9|v|} A \phi_{\max} \Theta_{\max}^5 \left\{ \left[2 + 2\rho + \rho^2 + \frac{1}{3}\rho^3 - \frac{2}{3}\rho^4\right] e^{-\rho} - \rho^{9/2} \mathrm{erfc}\left(\sqrt{\rho}\right) \right\}_{R_{\max}}^{R_{\min}} \ , \tag{16}$$

where, $\rho \equiv \frac{R}{\Theta_{\max}}$ and $\Theta_{\max} \equiv 8T/(3\pi\sqrt{2\pi}\phi_{\max}^2)$ is the thermal length-scale for the system which emerges from the calculation. An identical expression holds for $I_2$, but with $\phi_{\max}$ replaced by $\Delta_+ \equiv |\phi_{\max} - \phi_+|$, and $\Theta_{\max}$ replaced by $\Theta_{\max}^+ \equiv 8T/(3\pi\sqrt{2\pi}\Delta_+^2)$. The ratio $A/|v|$ can be obtained numerically, as in [10] or, in principle, analytically, although we leave it as a free parameter here.

FIG. 3. $\gamma_+(T_G)$ for the standard model.



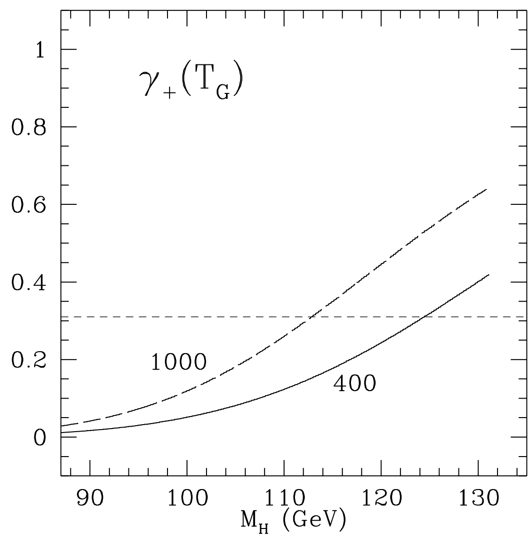

Using these expressions, we can estimate $\gamma_+(T_G)$ for the SM and for the MSSM. We plot the results in figures (3) and (4), respectively.

We can now refer back to figures (1)-(4) and analyze the different scenarios. There are two different issues that should be discussed: first, the mass range for which the condition $\langle\phi(T_G)\rangle/T_G \geq 1$ is satisfied; and second, how $\gamma(T_G)$ compares with the percolation threshold probability, $p_{\rm perc} = 0.31$. For the MSSM, the case of most immediate interest, $\langle\phi(T_G)\rangle/T_G \geq 1$ for $m_H^{\rm MSSM} \leq 116$ GeV (Figure 2), while $\gamma(T_G) = 0.31$ at $m_H^{\rm MSSM} = 124$ GeV, for $A/|v| = 400$ (Figure 4). Thus, in this scenario, the broken-symmetric phase does not percolate within the range of Higgs masses for which $\langle\phi(T_G)\rangle/T_G \geq 1$. The allowed mass range for baryogenesis is extended to at least $m_H^{\rm MSSM} = 116$ GeV. This is a lower bound because, as we remarked earlier, even if $\langle\phi(T_G)\rangle/T_G < 1$, it is possible for the condition $\langle\phi(T)\rangle/T \geq 1$ to be achieved at a lower temperature $\bar{T}$, as long as $\bar{T}$ is fairly close to $T_G$ (otherwise the transition would be completed before $\bar{T}$ and no net baryon excess would be produced). Thus, it is possible, in principle, to extend the allowed mass range even further than the value dictated by the condition $\langle\phi(T_G)\rangle/T_G > 1$, although a detailed analysis of the dynamics of the phase transition is required in order to make a more quantitative statement. An analogous calculation for the SM yields an allowed mass range extended to at least $m_H^{\rm SM} = 45$ GeV. We should comment on the choices for the values of $A/|v|$. The values for the prefactor of the subcritical bubble nucleation rate ($A$) or its shrinking velocity in a plasma ($v$) are not known. However, from results for critical bubble nucleation and expansion in the electroweak transition, it is reasonable to expect that $1 \leq A \leq 100$ and $0.005 < v < 0.5$. We are treating $A/|v|$ as a free parameter (fortunately, only the ratio of the two is relevant), and values between 100 and 1000 seem to be reasonable, judging from the ranges of $A$ and $|v|$. Hopefully, detailed simulations of weakly first order transitions could further restrict the range of $A/|v|$.

We have described how electroweak baryogenesis can proceed even if the phase transition is weakly first order. This idea is radically different to existing lore about such scenarios. In weak phase transitions, baryon number violating processes are sufficiently copious in the broken phase that any baryon excess is washed out at $T_c$. However, the important point is that the transition dynamics can be such that, at a lower temperature, when baryon number violation becomes exponentially suppressed in the broken phase (at $\bar{T}$), the transition may not yet have completed. There then remains the possibility for a sufficient baryon excess to be produced during the remainder of the transition. Thus, this approach may have important ramifications for electroweak baryogenesis. In this letter we have described how to estimate the relevant quantities in these scenarios analytically. In a later pub-

lication we will present a detailed semi-analytical and numerical approach in which we will obtain more precise values. This will involve taking into account the full form of the free energy, the effects of nonabelian gauge interactions, and an accurate analysis of the phase transition dynamics and freeze-out rates.

FIG. 4. $\gamma_+(T_G)$ for the MSSM. The two curves are for $A/|v| = 400$ (continuous line) and 1000 (dotted line).

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